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A study of spacetime distortion around a scattered recoiling D-particle and possible astrophysical consequences

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Abstract

We study a four-dimensional spacetime induced by the recoil of a D(irichlet)-particle, embeded in it, due to scattering by a moving string. The induced spacetime has curvature only up to a radius that depends on the energy of the incident string. Outside that region ('bubble') the spacetime is matched with the Minkowski spacetime. The interior of the bubble is consistent with the effective field theory obtained from strings, with non-trivial tachyon-like and antisymmetric tensor fields (in four dimensions the latter gives rise to an axion pseudoscalar field). The tachyonic mode, however, does not represent the standard flat-spacetime string tachyon, but merely expresses the instability of the distorted spacetime. Due to the non-trivial matter content of the interior of the bubble, there is entropy production, which expresses the fact that information is carried away by the recoil degrees of freedom. We also demonstrate that a particle can be captured by the bubble, depending on the particle's impact parameter. This will result in information loss for an external asymptotic observer, corresponding to production of entropy propotional to the area of the bubble. For the validity of our approach it is essential that the string length is a few orders of magnitude larger than the Planck length, which is a typical situation encountered in many D-brane-world models. A very interesting feature of our model is the emission of high-energy photons from the unstable bubble. As a result, the neighborhood of the recoiling D-particle defect may operate as a source of ultra-high-energy particles, which could reach the observation point if the source lies within the respective mean-free paths. This may have possible non-trivial physical applications, e.g. in connection with the observed apparent "violations" of the GZK cutoff.

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1 Introduction

It has been argued previously [1] that the recoil of a D-particle embedded in a four-dimensional spacetime (obtained from appropriate compactification of a higher-dimensional stringy spacetime) results in the following metric:

$$ds^2 = \frac{b'^2 r^2}{t^2} dt^2 - \sum_{i=1}^{3} dx_i^2 \ , \qquad r^2 = \sum_{i=1}^{3} x_i^2 \tag{1}$$

The above metric is derived by demanding appropriate Liouville dressing of the corresponding non-critical string, the non-criticality being due to the presence of the recoil world-sheet vertex operators [2], which are *relevant operators* in a world-sheet renormalization group sense. As discussed in [3], the dimensionless parameter b' depends on the energy of the incident string, E, and, in fact, represents the *quantum uncertainty* in momentum (in units of M_s) of the recoiling D-particle.

The spacetime (1) may be considered as a *mean field* result of appropriate resummation of quantum corrections for the collective coordinates of the recoiling D-particle. To lowest order in a weak string ($g_s < 1$) σ-model perturbative framework, such quantum fluctuations may be obtained by resumming pinched annuli world-sheets, which can be shown to exponentiate [3], thereby providing a Gaussian probability distribution over which one averages. The metric (1) is the result of such an average, and the eventual identification of the Liouville mode with the target time [4].

For low energies compared to string scale M_s, the parameter $b' = b'(E)$ is given by [3]:

$$b'(E) = 4g_s^2 \Big(1 - \frac{285}{18} g_s^2 \frac{E}{M_D}\Big) \tag{2}$$

where E denotes the *kinetic* energy of the recoiling D-particle, g_s is the string coupling, assumed weak, and $M_D = M_s/g_s$ is the D-particle mass, which is formally derived in the logarithmic conformal field theory approach from energy-momentum conservation [3].

It should be remarked at this point that we shall be working throughout this work with g_s small but *finite*. The limit $g_s \to 0$ will not be considered, given that when $g_s = 0$ the mass of the D-particle $M_D \to \infty$, and thus the recoil is absent, but on the other hand the curvature of the surrounding spacetime as a result of the immense mass of the D-particle should be taken into account. In that limit the scale $b'(E) \to 0$, and thus one can no longer consider distances sufficiently far away from the center of the infinite gravitational attraction so that the Schwarzschild curvature effects of the D particle could be ignored.

As one observes from (2), the value of b' decreases with increasing energy, and formally vanishes when the energy is close to M_s. We should note, however, that the above expression (2) pertains strictly to slowly moving strings, i.e. $E \ll M_s$. In general, for arbitrary energies (including intermediate ones, which we shall be interested in below), the precise expression for $b'(E)$ is not known at present. For our purposes, we shall assume that $b'(E)$ decreases with increasing E for all energies. This will be justified later on.

A crucial ingredient of the approach is the identification of target time with the Liouville mode ϕ [4]. In order for this procedure to be consistent, the resulting effective field theory must satisfy the appropriate σ-model conformal invariance conditions, which in a target-space framework correspond to appropriate equations of motion derived from a string-effective action. In this note we shall demonstrate that this is indeed the case, to lowest order in a Regge slope α' perturbative expansion. We shall also study some physically important properties of the spacetime (1). We shall argue that this spacetime is unstable, with the inevitable result of emission of high-energy radiation. Then we shall speculate on possible astrophysical applications of this phenomenon. Specifically, we shall argue that, as a result of the high-energy-photon emission from the unstable bubble, the neighborhood of the recoiling D-particle defect may behave as a source of ultra-high-energy particles, which can reach the observation point, provided that the defect lies at a distance from Earth which is within the respective mean-free paths of the energetic particles. This effect,

then, may have a potential connection with the recently observed [5] apparent 'violations' of Greisen-Zatsepin-Kuzmin (GZK) cutoff [6].

2 Dynamics of the Recoil spacetime

In four-dimensional spacetime, obtained by appropriate compactification of the higher-dimensional spacetime, where string theory lives, the string massless multiplet consists of a graviton field $g_{\mu\nu}$, a dilaton Φ and an antisymmetric tensor field $B_{\mu\nu}$, which in four dimensions gives rise, through its field strength $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$, to a pseudoscalar axion field b (not to be confused with the uncertainty parameter b'). The latter is defined as follows:

$$H_{\mu\nu\rho} = \frac{1}{\sqrt{-g}}\epsilon_{\mu\nu\rho\sigma}\partial_\sigma b \tag{3}$$

What we shall argue below is that the spacetime (1) is compatible with the equations of motion obtained from a string effective action for the above fields. Equivalently, these equations are the σ-model conformal invariance conditions to leading order in the Regge slope α'. We stress once again the fact that the spacetime metric (1) has been deived upon the non-trivial assumption that the target time is identified with the Liouville field [4], whose presence is necessitated by the recoil [1, 2, 3]. The fact, as we shall see, that this identification is consistent with the σ-model conformal invariance conditions to $\mathcal{O}(\alpha')$, is therefore a highly-non-trivial consistency check of the approach.

The components of the Ricci tensor for the metric (1) are:

$$R_{00} = -\frac{2b'^2}{t^2}\ , \qquad R_{ij} = \frac{\delta_{ij}}{r^2} - \frac{x_i x_j}{r^4}\ , \qquad i,j = 1,2,3. \tag{4}$$

The curvature scalar, on the other hand, reads:

$$R = -\frac{4}{r^2} \tag{5}$$

which is independent of $b'(E)$ and singular at the origin $r = 0$ (initial position of the D-particle). Thus, we observe that the spacetime after the recoil acquires a singularity. However, our analysis is only valid for distances r larger than the Schwarzschild radius of the massive D-particle, and hence the locus of points $r = 0$ cannot be studied at present within the perturbative σ-model appropach. It is therefore unclear whether the full stringy spacetime has a true singularity at $r = 0$.

The Einstein tensor $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ has components:

$$G_{00} = 0\ , \qquad G_{ij} = -\frac{\delta_{ij}}{r^2} - \frac{x_i x_j}{r^4} \tag{6}$$

The conformal invariance conditions for the graviton mode of the pertinent σ-model result in the following Einstein's equations as usual:

$$G_{\mu\nu} = -\mathcal{T}_{\mu\nu} \tag{7}$$

where $\mathcal{T}_{\mu\nu}$ contains contributions from string matter, which in our case includes dilaton and antisymmetric tensor (axion) fields, and probably cosmological constant terms (which will turn out to be zero in our case, as we shall see later on). As we shall also show, there are tachyonic modes necessarily present, which, however, are not the ordinary flat-spacetime Bosonic string tachyons. In fact, despite the fact that so far we have dealt explicitly with bosonic actions, our approach is straightforwardly extendable to the bosonic part of superstring effective actions. In that case, ordinary tachyons are absent from the string spectrum. However, our type of tachyonic modes, will still be present in that case, because as we shall argue later, such modes simply indicate an instability of the spacetime (1).

We find it convenient to use a redefined stress energy tensor $\mathcal{T}'_{\mu\nu} \equiv \mathcal{T}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{T}^{\alpha}_{\alpha}$, in terms of which Eisntein's equations become:

$$R_{\mu\nu} = -\mathcal{T}'_{\mu\nu} \tag{8}$$

The stress tensor $\mathcal{T}'_{\mu\nu}$ for the case of tachyon and axion fields reads:

$$\mathcal{T}'_{\mu\nu} = \partial_\mu T \partial_\nu T + \partial_\mu b \partial_\nu b - g_{\mu\nu} V(T) \tag{9}$$

where $V(T)$ is a potential for the tachyonic mode T. The fact that the axion field b does not have a potential is dictated by the abelian gauge symmetry of string effective actions, according to which they depend only on the antisymmetric-field strength $H_{\mu\nu\rho}$ and *not* on the field $B_{\mu\nu}$. Below we shall show that indeed the field T acquires a tachyonic mass, which however, in contrast to the flat-space time Bosonic string theory tachyons, depends on the parameter $b'(E)$.

In addition to (8), one has the conformal invariance conditions for the tachyon and axion fields, to $\mathcal{O}(\alpha')$:

$$\partial^2 T = -V'(T) \ , \qquad \partial^2 b = 0 \tag{10}$$

where the prime denotes differentiation with respect to $T(x_i, t)$.

A solution to (8), (10) is given by:

$$T(x_i, t) = \ln r \ , \qquad b(x_i, t) = b'(E) \ln t \tag{11}$$

provided that the tachyon potential $V(T)$ is:

$$V(T) = -\exp\Big(-2T(r)\Big) \tag{12}$$

Naively, if the solution is extended to all space, we observe that the matter diverges logarithmically. To remedie this fact we restrict the above solution to the range $r \le t/b'(E)$, and thus we enclose it in a *bubble* of time-dependent radius $t/b'(E)$. Outside the bubble we demand the spacetime to be the flat Minkowski spacetime, and thus the above upper limit in r, $t/b'(E)$ is the locus of points at which the temporal component of the metric (1) becomes unity, and this allows an appropriate matching of the interior and exterior geometries. As we shall see later on, a non-trivial consistency check of this matching will be provided *dynamically* by an explicit study of the scattering of test particles off the bubble. In this way the phenomenologically unwanted tachyon and probably axion fields (obtained from the antisymmetric tensor field of the string) are confined inside the bubble of radius $t/b'(E)$ (cf. figure 1).

In arriving at the above solution we have restricted ourselves to $\mathcal{O}(\alpha')$ because we have ignored terms of higher order in curvature R. To justify such an approximation it suffices to note that the ratio of the leading terms to the next to leading ones is:

$$(R/M_s)^2/R = \mathcal{O}\Big(b^2\Big) = \mathcal{O}\Big(g_s^2(1 - g_s^2 \frac{285}{18} \frac{E}{M_D})\Big) \tag{13}$$

provided one does not approach the singularity at $r = 0$, which is in fact consistent with the regime of validity of the logarithmic conformal field theory [2, 3], i.e. distances far away from the defect, and times long enough after its collision with the string. This implies that our analysis is restricted near the boundary of the bubble, which will be sufficent for our purposes in this work. Since the string coupling is assumed weak, it is evident from (13) that the approximation of neglecting the higher-curvature terms is satisfactory near the boundary of the bubble. Then, from the decreasing behaviour of $b'(E)$ with increasing energy, which, as mentioned previously, is assumed here even for intermediate energies, it follows that this approximation becomes even better for higher energy scales appropriate for the early stages of the universe.

A second important remark concerns the fact that in the analysis leading to the metric (1) we have treated the spacetime surrounding the D-particle defect as initially (i.e. before the collision

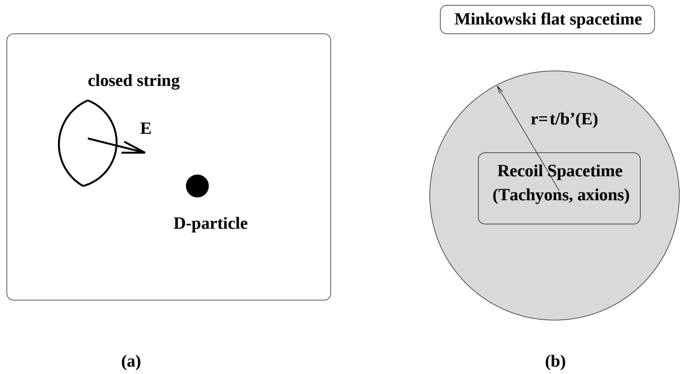

Figure 1: (a) Scattering of a closed string mode (of energy E and non-trivial momentum) off a D-particle embedded in a four-dimensional spacetime (obtained by compactification of a higher-dimensional stringy spacetime). (b) after the recoil: formation of effective bubble, in the interior of which the tachyonic modes and axion fields are confined. The exterior geometry is the flat Minkowski.

with the string) flat. However, even the initially at rest D-particle, being a very massive one of mass $M_D = M_s/g_s$ would naturally curve the spacetime around it, producing a Schwarzschild radius $r_S = \ell_P^2/g_s\ell_s$, where ℓ_P denotes the four-dimensional Planck length and $\ell_s = 1/M_s$ the string length. From our discussion above, the radius of the bubble of figure 1 is $r_b = \ell_s/g_s$. For consistency of our approach, the approximation of treating the spacetime as initially flat implies that we work in distances considerably larger than the Schwarzschild radius, so as the general relativistic effects due to the mass of the D-particle could be safely ignored. This implies that the radius of the bubble must be considerably larger than r_S, i.e.

$$1 \gg r_s/r_b = \Big(\ell_P/\ell_s\Big)^2 \tag{14}$$

From the modern view point of string/D-brane theory, the string length may not be necessarily of comparable order as the Planck legth, but actually could be larger. Thus, the above condition seems consistent.

Notice also that the matching with the flat Minkowski spacetime in the exterior geometry is possible because the matter energy density $\mathcal{T}_{00}$ and energy flow $\mathcal{T}_{0i}$ in the interior of the bubble of figure 1 are *both zero*:

$$\mathcal{T}_{00} = \mathcal{T}_{0i} = 0 \tag{15}$$

and thus there is no radiation coming out of or flowing into the bubble.

We next compute the mass squared of the field T. To this end, we shall consider the fluctuations δT of the tachyonic mode T around the classical solution $T_{\rm cl}$ (11) in the interior of the bubble, but close to its boundary, where the spacetime approaches the Minkowski flat geometry. From (10) we then have:

$$\partial^2 \delta T - 4e^{-2T_{\rm cl}}\delta T\Big|_{r\to t/b'(E)} = 0 \tag{16}$$

from which we obtain a mass squared term of the form:

$$m^2 = -4\frac{b'^2(E)}{t^2}\ . \tag{17}$$

The negative value indicates, of course, the fact that the field T is tachyonic, but the interesting issue here is that the induced mass depends on the parameter $b'(E)$, and hence on the initial energy data of the incident string.

A remark we would like to make at this point concerns the time-dependence of the mass (17). As one observes from (17), the tachyon field will eventually disappear from the spectrum (as it becomes massless) asymptotically in time t. This fact comes from the specific form of the metric (1). However, as argued in [1] quantum effects will eventually stop the expansion of the bubble and may even force it to contract. In this sense, the value of the mass of the tachyonic mode (17) will remain finite and negative, and will never relax to zero.

An equivalent way of seeing this is to observe that the time t in the temporal component of the metric (1) may be absorbed by a redefinition of the time $t \to t' = \ln t$. In that case, the metric reads:

$$ds^2 = {b'}^2(E) r^2 d{t'}^2 - \sum_{i=1}^{3} dx_i^2 \tag{18}$$

Under this redefinition, the bubble solution remain, but this time the bubble appears to be independent of time, with its radius being $r = 1/b'(E)$, and the mass squared of the tachyon being $m^2 = -4{b'}^2$. From that we observe that the tachyon mass remains $b'(E)$-dependent and finite. This argument supports the fact that even in the initial coordinate system (1) the time t cannot be such so as to eliminate the $b'(E)$ dependence of the tachyon mass. This system of coordinates corresponds to a frame in which the bubble appears static, hence it corresponds in some sense to a *comoving frame*. Therefore the mass $m^2 = -4{b'}^2(E)$ is the rest mass of the tachyonic mode.

Some clarifying remarks are in order at this point concerning the nature of the tachyonic mode T. As we have just seen its mass is dependent on the uncertainty scale $b'(E)$ of the D-particle, and hence is proportional to the string coupling g_s (cf. (2)). For this reason this tachyonic mode should be distinguished from the standard tachyon fields in flat-spacetime free Bosonic string theory. In fact, in our approach this tachyonic mode expresses simply the *instability* of the *bubble* configuration, and will be present even in superstring effective field theories.

As a result, from such consideration one may obtain an average *lifetime* τ for the bubble:

$$\tau = \frac{1}{2b'(E)} \tag{19}$$

In our approach we prefer to work with the initial form of the metric (1), implied directly by the logarithmic conformal field theory approach to recoil [2, 1], which defines a natural frame for the definition of the observable time. In this approach, the presence of the recoil degrees of fredom after a time, say, $t = 0$ imply a breaking of the general coordinate invariance by the background, and also an irreversibility of time [1]. This comes from the fact that in our problem, time is a world-sheet renormalization group parameter (Liouville mode) [4], which is assumed irreversible, flowing towards a non-trivial infrared fixed point of the world-sheet renormalization group of the Liouville σ-model [7, 1]. In this frame the mass of the tachyon appears time dependent, because, as we shall discuss below, the frame is not an inertial one, given that the spacetime of the bubble is a Rindler *accelerated spacetime*.

3 Thermal Effects and Radiation from the Bubble Spacetime

To show this we consider the metric (1), pass onto spherical polar coordinates (r, θ, ϕ), and fix the angular part for convenience, as this does not modify the concusions. We then perform the following coordinate transformations (from now on we work in units $\ell_s = 1$):

$$R = r\cosh(b'(E)\ln t)\ , \qquad T = r\sinh(b'(E)\ln t), \tag{20}$$

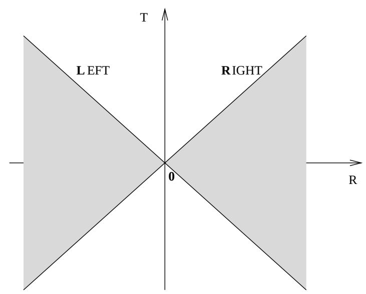

Figure 2: Rindler wedge spacetime arising from the recoil of a D-particle, embedded in a four-dimensional spacetime, due to its scattering with a closed string. The set of wedges (LEFT and RIGHT) describe the spacetime for $t > 0$.

The transformation maps our space time to the right Rindler wedge (R) depicted in figure 2. The left wedge (L) is described by similar transformations up to a minus sign. In the (R, T) coordinates the line element becomes:

$$ds^2 = dT^2 - dR^2 - (R^2 - T^2)d\Omega^2 \tag{21}$$

where $d\Omega$ is the conventional solid angle.

From the above spacetime, we observe that for distances $R \gg T$ one recovers the flat Minkowski spacetime, whilst for distances $R \sim T$ one obtains the *bubble* spacetime. Notice that $R^2 - T^2 = r^2$, and the interior of the bubble is defined by $r \leq 1/b'(E)$ in comoving coordinates.

An observer comoving with the expanding bubble, placed at position r, is accelerated with respect to the (R, T) frame, with proper accelration $1/r$. According, then, to the standard analysis of accelerated observers [8], such an observer sees the Minkowski vacuum (in the (R, T) coordinates) as having a non-trivial *temperature* $T_{\rm bubble}$

$$T_{\rm bubble} = \frac{1}{2\pi r}, \qquad 0 < r < 1/b'(E) \tag{22}$$

The temperature for the inertial Rindler observer T_0 is:

$$T_0 = \sqrt{g_{00}} T_{\rm bubble} = \frac{b'(E)}{2\pi} \tag{23}$$

The presence of temperature is expected to imply a non-trivial proper entropy density s. For a massless scalar field, in our case the axion b, the entropy $S = \int d^3x \sqrt{-g} s$ in four spacetime dimensions is given by:

$$S = \int d^3x \sqrt{-g} \frac{4}{3} \frac{\pi^2}{30} T_0^3 \tag{24}$$

From the bubble spacetime (1) one then obtains:

$$S = \int d^3x\sqrt{-g}s = 4\pi b'(E)\int_0^{1/b'(E)} dr\ r^3\frac{4}{3}\frac{\pi^2}{30}\frac{b'^3(E)}{8\pi^3} = \frac{1}{180} \tag{25}$$

Thus the bubble carries non-trivial entropy, which turns out to be independent of $b'(E)$. The reader should not be alarmed by the apparent volume independence of the entropy, which at first sight would seem to contradict the fact that the entropy is an extensive quantity. In fact, there is no contradiction in our case, since there is only one scale in the problem, $b'(E)\ell_s$, and the volume of the bubble is itself expressed in terms of this scale.

The presence of entropy production after the recoil implies *loss of information* which can be understood as follows: one starts from a pure state of a string striking a D particle. There is no entropy in the initial configuration. After the strike, the D-particle recoils, and because it is a heavy object it distorts the spacetime around it, producing the bubble phenomenon via its recoil excitation degrees of freedom. Due to the finite lifetime of the bubble, the entropy (25) will be released to the external space, implying information encoded in the recoil degrees of freedom, which are unmeasurable by an asymptotic observer. This picture is consistent with the loss of conformal invariance of the underlying σ-model, and the irreversible world-sheet renormalization-group flow of the recoiling system, as discussed in [1, 4], upon the identification of the Liouville mode with the target time.

It should be remarked at this point that the entropy (25) pertains strictly to scalar fields that live *inside the bubble*. There is no crossing of the surface of the bubble by the interior fields in our construction, at least classically (as we shall discuss below there is a quantum-mechanical escape probability). On the other hand, it should be noticed that for a field which lives in the *exterior* of the bubble, there appears to be loss of information in the sense that the exterior particle degrees of freedom may enter the bubble and be captured, as we shall discussed in the next section. Thus, an asymptotic observer, far away from the bubble, will necessarily *trace out* such (unobserved) degrees of freedom in a density matrix formalism, and in this sense the resulting entropy, pertaining to such degrees of freedom, will be proportional to the *area* of the bubble and *not its volume* [9]. We shall come back to this important point, in connection with emitted radiation from the bubble later on.

The presence of temperature T_0 (23) implies the emission of radiation from the bubble (see figure 3), which can be read off from the Stefan-Boltzman law σT_0^4, $\sigma = \pi^2/60$ (in units $\hbar = c = k_B = 1$). Given that the area is $4\pi b'^{-2}(E)$, and that the lifetime of the bubble is estimated from (19), one observes that during the life time τ, the following amount of energy is released in the form of radiation:

$$E_{\rm rad} \sim 4\pi b'^{-2}(E)\tau\sigma T_0^4 = \frac{1}{480\pi}b'(E)M_s = \frac{g_s}{480\pi}b'(E)M_D \tag{26}$$

It is interesting to observe that the same amount of energy represents the *thermal* energy of the axion field in the interior of the bubble:

$$E_{\rm th/axion} = \int d^3x\sqrt{-g}\frac{\pi^2}{30}T_0^4 = \frac{g_s}{480\pi}b'(E)M_D \tag{27}$$

From (2), and taking into account that in the effective field-theory limit we are working here, $E < M_s$, and that $g_s \ll 1$ in our weak string framework, one observes that the energy $E_{\rm rad} = E_{\rm th/axion} < M_D$.

From energy conservation then, which, notably, is shown to be valid rigorously in the context of our logarithmic conformal field theory (stringy) recoil framework [3], one obtains:

$$E_{\rm in} + M_D = M_D + E + E_{\rm th/axion} \tag{28}$$

where $E_{\rm in}$ denotes the total energy of the *incident* particle/string. From this, one thus sees that there is a *threshold* for bubble formation:

$$E^{\rm threshold} = E_{\rm rad} = E_{\rm th/axion} \tag{29}$$

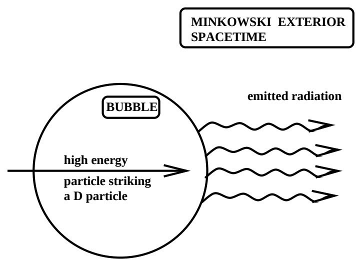

Figure 3: Emitted radiation from the unstable bubble. The radiation is not isotropic, but most of it will be emitted in the forward direction, parallel to that of the incident high-energy particle.

From these considerations, one observes that the radiation energy *will not cause any mass loss* of the D-particle, since all the thermal axion energy accounts for that. Hence, despite the instability of the bubble, the stability of the D-particle defect is not affected. This will be important for our physical applications, to be discussed later on.

Notice that the energy release (26) cannot make up for the maximum of the thermal energy expected from Wien's law $\lambda T_{\rm max} = {\rm const}$, where λ the wavelength of radiation. Thus the resulting photon spectrum is *not thermal*, and hence one can only get an estimate for the energies of the emitted radiation. The alert reader might then object to our previous use of the black body (or, in general, equilibrium) laws. In fact their use is indicative, and they can only give qualitative results (e.g. in our case we only obtain a tail of the thermal distribution).

To recapitulate, the physics behind the above properties can be summarized as follows: one needs a highly-energetic incident particle of energy $E_{\rm in} > E^{\rm threshold}$, which strikes a D-particle, and forms a bubble; the bubble radiates an amount of energy $E^{\rm rad}$ (26) distributed appropriately among the various photons. The emitted radiation will not be isotropic as a result of (spatial) momentum conservation (see figure 3). This will be useful in physical applications. The fact that a particle, entering and being captured by the bubble, will cause the emission of radiation from the bubble is nicely related to the existence of non-zero entropy measured by an asymptotic observer. This phenomenon is thus not dissimilar to the Hawking process of an evaporating black hole, although in our case the bubble is *not* a black hole, neither the D-particle evaporates, as we discussed above. This picture is in agreement with the non-equilibrium Liouville string framework [1], on which the approach is based.

4 Motion of particles in the bubble spacetime

We shall now analyze the motion of a particle in the bubble spacetime (1). For convenience we shall work at the equator of the three sphere (fixed angle $\theta = \pi/2$). The lagrangian of the particle

in the background spacetime (1) in comoving coordinates (t', r, ϕ) is:

$$\mathcal{L} = \frac{1}{2}\Big(\frac{ds}{d\lambda}\Big)^2 = \frac{b'^2(E)r^2}{2}\Big(\frac{dt'}{d\lambda}\Big)^2 - \frac{1}{2}\Big(\frac{dr}{d\lambda}\Big)^2 - r^2\frac{1}{2}\Big(\frac{d\phi}{d\lambda}\Big)^2 \tag{30}$$

Expressing the Lagrangian in terms of the conserved angular momentum $L = r^2(d\phi/d\lambda)^2$ and energy $E_{\rm in} = b'^2(E)r^2\frac{dt'}{d\lambda}$ we obtain:

$$\frac{E_{\rm in}^2}{2b'^2(E)r^2} - \frac{1}{2}\Big(\frac{dr}{d\lambda}\Big)^2 - \frac{L^2}{2r^2} = \frac{\mu^2}{2} \tag{31}$$

where $\mu = 0$ for massless particles (e.g. photons), and 1 for massive particles (in which case the quantities L and $E_{\rm in}$ are the corresponding quantities per unit mass).

Writing the equation of motion as:

$$\frac{1}{2}\Big(\frac{dr}{d\lambda}\Big)^2 + \frac{\mu^2}{2} = \frac{E_{\rm in}^2/b'^2(E) - L^2}{2r^2} \tag{32}$$

we can see that the impact parameter $L/E_{\rm in}$ must be smaller than $1/b'(E)$, for the equation to make sense. This means that if $L/E_{\rm in} > 1/b'(E)$, then the particle will *necessarily* travel *outside* the bubble spacetime, which is thus a dynamical consistency check of our matching assumptions that the spacetime in the region $r \geq 1/b'(E)$ is the flat Minkowskian spacetime. Such outside particles will not be affected by the presence of the bubble, and their trajectory will be undisturbed, that predicted by special relativistic dynamics.

Below we shall study now the case of impact parameters $L/E_{\rm in} < 1/b'(E)$, for massless and massive particles. In such a case, from (32) the massless particle equation of motion reads:

$$r = r_0 \exp\Big(\pm\phi\sqrt{\frac{E_{\rm in}^2}{b'^2(E)L^2} - 1}\Big) \tag{33}$$

where in the case of an incoming photon (from outside the bubble) $r_0 = \frac{1}{b'(E)}$, and we have taken only the $(-)$ sign, because this is the only consistent choice. This shows that the massless particle will be *captured* inside the bubble.

From (32) one observes that, in the case of a massive particle, there exist values of energy and angular momentum such that the particle can escape the bubble spacetime. This happens if the radial velocity on the boundary (as the particle attempts to escape) is non-zero, which implies (we have re-instated the units of M_s for clarity):

$$E_{\rm in}^2 - L^2 b'^2(E) M_s^2 > M_s^2 \tag{34}$$

This demonstrates that only highly energetic particles with energies much higher than M_s can escape the bubble spacetime.

Once such a particle is electrically charged, its non-uniform (spiral) motion inside the bubble will cause the emission of *radiation*. The latter will continue to carry angular momentum and energy of roughly the same order as that of the particle. Because the emission is now taken place at $r_0 < 1/b'(E)$ the positive sign in the exponent of equation (33) is also allowed, implying an escape possibility for the emitted photons (cf. figure 4). In addition, as we shall show in the next section, the bubble has a non-trivial (thermal) refractive index, and thus behaves as a medium. If there is a beam of charged particles entering the bubble within its short life time, then these particles will experience the (suppressed) phenomenon of *transition radiation* [10], i.e. the emission of photons accompanying an electrically-charged particle when crossing the interface separating two media with different refractive indices (in our case, the interior of the bubble and the exterior Minkowski spacetime). A fraction of this radiation will also escape the bubble. Such phenomena, if true, will imply excess of photons accompanying the charged particle. We shall present a more

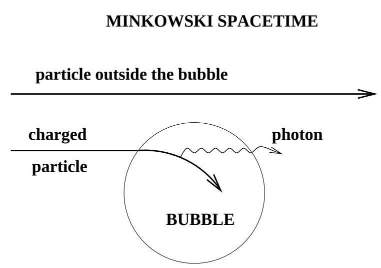

Figure 4: Emission of photons due to non-uniform motion of a charged particle inside the bubble. The radiation can escape the bubble.

detailed discussion of these issues, and their potential experimental consequences, in a forthcoming publication.

It should be noticed that, if the condition (34) is satisfied, then the particle is *deflected* by an angle $\Delta\phi$ which can be computed in a standard way to be:

$$\Delta\phi = \pi - \frac{2}{\sqrt{\frac{1}{\rho^2 b'^2(E)} - 1}} \mathrm{arccosh}\Big(E_{\rm in}\sqrt{1-\rho^2 b'^2(E)}\Big) \; , \qquad \rho = L/E_{\rm in} = \text{impact parameter} \quad (35)$$

From this we observe that, for fixed impact parameter ρ, highly energetic particles will not be deflected much, as should be expected.

As a final comment we mention that the scattering cross section $\sigma(E)$ for energies and/or angular momenta that violate the condition (34), which are of physical interest, is given by:

$$\sigma(E) = \pi b'^{-2}(E) \quad (36)$$

From the results of ref. [3], which are valid strictly only for low energies, we observe that $b'(E)$ decreases with increasing energy E (2). One would expect intuitively that strings with higher energy would cause larger distortion of the spacetime surrounding the recoiling D-particle defect. This point of view is supported by the above results if one extends the behaviour of $b'(E)$ encoded in (2) to intermediate energies as well, and in fact to all energies (up to Planckian), because in that case the distortions of spacetime caused by strings with higher energy will correspond to formation of bubbles with bigger radii $1/b'(E)$.

From (36) we observe that higher energy strings would correspond to larger cross sections. The important point to notice is that the cross section is non zero even for zero energy. This stems from (2), and is associated with the fact that $b'(E)$ is essentially a *quantum uncertainty* in the momentum of the D-particle [3], which is not zero even for vanishing incident energy E. Hence, the spatial uncertainty of the position of the D-particle, which is associated with the cross section, is not zero, but is bounded from below by the Heisenberg uncertainty principle, which explains

naturally the non-zero minimum value of $\sigma(E = 0)$. It also explains the increasing behaviour of the cross section with increasing energy, given that the higher the energy is, the larger the uncertainty is expected to be.

5 Quantum Electrodynamics Effects inside the Bubble and Refractive Index for Photons

So far our considerations have been classical. It is in this sense that we demonstrated capture of photons (and, in general, massless particles) in the interior of the bubble. The presence of finite temperature (23) will create a non-trivial (thermal) vacuum, with broken Lorentz symmetry inside the bubble. In that case, it is known [11] that the effective velocity of light, defined by the quantum propagator of photons, is modified in accordance with the fact that the finite temperature effects provide the notion of a medium.

Specifically, the dispersion relation for a particle of mass m in the non-trivial vacuum at temperature T is: $E^2(p) = p^2 + f(p, T, m)$, where E is the energy and p the momentum, and the function f encodes the quantum effects of the vacuum polarization. The group velocity of the particle is then given by $v = \partial E(p)/\partial p$, and in general depends on p.

In the case of photons in a non-trivial quantum electrodynamical vacuum, the function f can be computed, to one loop, from the vacuum-polarization graph of the photon [11]. The latter is of order $f \sim T^2 e^2$, where e is the electron charge [1].

In our case, the induced temperature (23) is much larger than the incident momentum p of the photons. From the results of [11] in this case, the effective velocity of the photons inside the bubble is (in units of speed of light *in vacuo* $c = 1$):

$$v \sim \frac{p}{eT_0} \sim \frac{p}{eb'(E)} \tag{37}$$

where p is related to the energy E of the photon via the (modified) dispersion relation. The photon in this case becomes *effectively* massive, with mass $\mu \sim eb'(E) \neq 0$. It is interesting to note that, due to the extreme temperature effects, $p \ll T_0$, the resulting photon is considerably slowed down inside the bubble to non-relativistic velocities.

From our earlier discussion on massive particle trajectories inside the bubble, and the fact that the quantum effects result in an effective photon mass, one is tempted to consider the possibility of a photon escaping the bubble. However, from the condition (34) and the induced photon mass $\mu \sim eb'(E)$, it becomes evident that there is *no such possibility*.

Nevertheless, there is a non-zero probability of *quantum tunneling* through the potential barrier. In this sense, part of the classically captured (incident) photons can escape. Combined with the thermal slowing down (cf. (37)) of all photons inside the bubble, then, this will result in the appearance of *delays* in the respective arrival times of photon beams from distant astrophysical sources in areas where there are D-particles. Because these delays will be associated with only part of the photon beam, the final effect will appear as a fluctuation in the arrival time.

The associated delay for a single photon, which passes through a region in space where there is one D-particle, and is assumed to escape through tunelling, is estimated to be:

$$\Delta t \sim \frac{1}{b'(E)v(E)} = \frac{e}{p} \tag{38}$$

where $\frac{1}{b'(E)}$ is the radius of the bubble, e is the electron charge, and this formula is applicable for $p \ll M_s$. For velocities $p \sim M_s$ there are modifications, which however are not of interest to us here.

The existence of delay effects that depend on the energies of the photons bares some resemblance to quantum spacetime effects, associated with induced refractive index for photons,

[1] Note that one can extend these results to incorporate all known particles in the standard model [11].

diuscussed in [12]. As in those works, so in the present model there appears to be a non-trivial refractive index $n(E) = 1/v(E)$ inside the bubble. However, there is an important difference, in that here this is due to conventional quantum electrodynamical thermal effects. Because of this reason, the induced refractive index (37) is reduced with increasing photon energies, in contrast to the effects of refs. [12], where the quantum spacetime induced refractive index appears to increase with energy. However, the reader should bear in mind that in our model the existence of temperature is due to quantum stringy effects [1, 3] associated with the recoil of the D-particle. In this sense there is, in our approach, a notion of quantum gravity, in similar spirit to the work of [12].

Unfortunately, for a very dilute gas of D-particles the maximal delay (38), corresponding to the less energetic observable photons (e.g. infrared background of energies 0.025 eV) is very small, or order 10^{-11} s. Nevertheless, we should notice that if one considers photon sources from distant astrophysical objects that are at distances corresponding to cosmological redshifts $z > 1$, then in those areas of the relatively early universe, the density of D-particles might have been higher. One then can get an upper bound of such densities by considering the effect (38) as lying inside the present experimental errors. We hope to come to a more systematic analysis of such effects in a forthcoming publication.

6 Discussion and possible physical applications on the GZK cutoff

In this work we have studied the formation of bublles as a result of scattering of closed strings off D-particles embedded in a four dimensional spacetime. Such configurations may be thought of as a trivial case of intersecting branes, provided one adopts the modern view point that our four-dimensional world is a $D3$-brane. In this sense, the string scale M_s, which enters our calculations, is not necessarily the same with the four-dimensional Planck scale, and as we have seen this played an important rôle in our analysis, as it allowed us to work at distances sufficiently far from the Schwarzschild radius of the D-particle, for finite g_s string couplings.

An interesting feature of our approach is the entropy production, which we associated with information carried away by the recoil degrees of freedom. This latter feature may have cosmological implications for mechanisms of entropy production in the early universe, where we expect the density of D-particles to be significant, and hence the probability of scattering with closed strings important. The fact that highly energetic charged particles can escape the bubble, with the simultaneous release of radiation, which can also escape and thus is in principle observable, is interesting, and might imply important phenomenological constraints on the order of the density of the D-particles in the Universe today. We plan to present a systematic study of such issues in a forthcoming publication.

Another comment we wish to make concerns the impossibilty of the extension of the above analysis, with the specific choice of fields, to higher-dimensional spacetimes, of spacetime dimension $d > 4$. This comes about because, under the simplest form for the tachyonic mode $T = \ln r$, consistent with Einstein's equations to order $\mathcal{O}(\alpha')$, one obtains the following form for the tachyonic-mode potential $V(T) \sim 1/r^2$. On the other hand, the equation of motion for the mode itself, consistent with the above form for T, demands that $dV(T)/dT = -(d-2)V(T)$. Clearly, this is consistent only *in* d*=4* spacetime dimensions. Hence, despite the fact that in our approach we have restored Lorentz invariance, we still obtain a special rôle of $d = 4$, as in the Lorentz violating scenario of [1], where the sacrifice of Lorentz invariance in the sense of an explicitly space-dependent vacuum energy, lead also to a selection of $d = 4$. If this feature survives the inclusion of the complete string matter multiplets, something which is not clear to us at present, then, it might imply that a *recoiling* Liouville D-particle cannot be embedded in (intersect with) a target spacetime (viewed itself as a D-brane), consistently with the σ-model conformal invariance, unless its dimensionality is $d = 4$. At present we consider the issue only as a mathematical curiosity of the specific effective field theory at hand, and we do not attribute it further physical

significance. However, surprises cannot be excluded.

A final comment we would like to make concerns possible physical applications of the above phenomena. The emitted radiation from the bubble spacetime may find important astrophysical applications as can be seen by the following simplified scenario: there is a rare distribution of D-particles in the inflated universe (although their density was much higher in the early universe). Due to this rare distribution, the concept of isotropy is not applicable. It is therefore possible that an isolated D-particle lies between Earth and a distant galaxy. As the galaxy emits particles, some highly energetic and weakly interacting ones, such as neutrinos, strike the D-particle and induce the formation of a bubble. The latter then emits radiation in the way explained above. The emitted photons, in the direction of the incident particles, will be highly energetic, of typical energy $b'(E)$. In general, such photons will interact with the background photons of either the microwave background radiation, or the infrared background, to yield, say, e^+e^- pairs. Because of this, if the D-particle lies far away from Earth, outside the average mean free path of such photons, the latter will not arrive on Earth. However, one may imagine a situation in which the isolated D-particle lies within the above mean free path distance, which in the case of photons interacting with the infrared background is estimated to be of order of a few Mpc [13]. Then, the weakly interacting incident particles, that trigger the phenomenon of bubble formation, e.g. neutrinos, will arrive at the location of the D-particle(s) undisturbed. In that case, the emitted high-energy radiation will reach the observation point, and in this sense the recoiling D-particle bubble *constitutes a novel and relatively nearby source of highly energetic photons*. Such scenaria may have applicability to the recently observed highly energetic 30 TeV photons that seem to violate the so-called GZK cut-off [13, 5].

In a similar way, one can also extend the above discussion to incorporate charged particles. Consider, for instance, a beam of protons emitted by a galaxy lying at cosmological distances, whose energies do not exceed the GZK cut-off, and hence they arrive undisturbed until the point where a D-particle defect lies. One of the protons will then strike the D-particle and create a bubble spacetime. As discussed previously, the proton will be captured inside the bubble, since its energy does not satisfy the escape condition (34), as being less than M_s [2]. The bubble will then radiate very high energy photons, which can interact with the remaining protons in the beam, that fly outside the bubble, to create, say, protons and pions *etc.* The protons (or, in general, the particles) that emerge from such interactions will then be very energetic, and it is conceivable that their energies can be of the order of the observed [5] 3×10^{20} eV. In this way, the region around the recoiling D-particle defect acts as a source of ultra-high-energy cosmic rays, and if one assumes, as before, that the defect lies within the mean free path of a proton (from Earth), this can easily explain the observed apparent "violations" of the GZK cut-off [5].

If the above scenario survives, it may then imply that there is no actual violation of Lorentz symmetry that is responsible for the phenomenon, as claimed by a number of authors [14], since in our bubble spacetime there is no such violation (except the trivial one due to temperature effects inside the bubble). To put it in simple terms, in our scenario, the source of the ultra-high-energy cosmic rays is in the neighborhood of the D-particle defect, which may lie at a much closer distance from Earth than one naively thought. A more detailed discussion of these speculative scenaria will constitute the topic of a forthcoming publication. Of course, it goes without saying that one cannot exclude the possibility that a peculiar combination of phenomena, involving the model discussed here in conjunction with both conventional and unvonventional (spacetime foam) physics [12, 14], might actually lie behind such extreme astrophysical phenomena.

Admittedly, the above ideas are very speculative, and indeed may have nothing to do at the end with real Physics. However, the mathematical and logical consistency of such unconventional effects seems, at least to the authors, convincing enough so as not to discard them immediately. It is actually very intriguing that stringy defects, which at first sight are not expected to play any rôle in low-energy physics, may actually be responsible for some extreme astrophysical phenomena, whose observation became only recently possible, as a result of the enormous technological

[2]Here we assume that M_s exceeds the GZK cut-off $10^{19}\text{eV} = 10^{10}$ GeV. If this is not the case, and one has a lower M_s, then such massive particles may escape. At any rate, our discussion does not depend upon this fact.

advances in both terrestrial and extraterrestrial instrumentation. It is always intellectually challenging, but also expected intuitively in some sense, to think of the vast Universe as being the 'next-generation' Laboratory, where ideas on the quantum structure of spacetime may be finally subjected to experimental tests in the not-so-distant future.

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